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Optimizing Mean-Variance Portfolio with Uncertainty Model

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The mean-variance optimization by Markowitz (1959) assumes that investors know the probability distribution of asset returns. However, in behavioural finance perspective, the probability distribution is uncertain because it depends on the individual investor's subjective belief. This study then introduces the portfolio with uncertainty in variance and the uncertainty in both mean and variance models. Both approaches are used to investigate the equity home bias puzzle by taking one asset as a domestic market, and the rest are foreign markets. The result shows that a familiar asset is more attractive than an unfamiliar asset under the uncertainty in variance. Nevertheless, when investors face the uncertainty in mean and variance, their allocation is similar to Markowitz's portfolio.

JEL Codes: G02 and G11

1. Introduction

Owing to unknown return and variance distributions in practice, the expected utility model by von Neumann and Morgenstern is not appropriate to explain the decision making of investor. As a result of this, many models such as Choquet expected utility and Max-min expected utility have been suggested to describe this behaviour. Max-min expected utility model argues that investors maximise the worst case scenarios under an uncertainty world (Gilboa and Schmeidler 1989). This model is later applied in the multi-prior technique by Garlappi *et al.,* (2007). The pioneering study on the uncertainty set in the optimal model shows that optimal weight from the uncertainty model deviates from the classical optimal weight. For example, the optimal weight will be depended on familiarity toward assets. Boyle *et al.*, (2012) confirms this theory by using a simulation to show how the weight of portfolio will change if there is an uncertainty in mean. Notwithstanding, their model is not concerned a situation under uncertainty in variance, and both mean and variance. As a result of this, the motivation of this paper is to making the model more realistic by optimizing the mean-variance portfolios under uncertainty in mean and variance. To enlighten investors to adjust their investment strategy with uncertainty, it is of interest to extend the multi-prior technique model to study indepth.

According to the previous model, this paper will extend to the uncertainty model by constructing two portfolios with three assets. For empirical results, the first part is to study the relationship between ellipsoidal uncertainty level and the optimal weight, in an uncertainty situation. Without overlooking the risk aversion coefficient of individual preference, the second part is to study how the risk aversion coefficient affects portfolios. The last section compares the out-of-sample performance between the classical model and the portfolio model with uncertainty.

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This research will contribute to two main areas: investment management and behavioural finance. As for the first area, because I employ real data to prove the theoretical framework, potential implications of my research may lead traders to reconsider their model. As for the behavioural finance perspective, this paper attempts to improve the traditional investment model with behavioural science, emphasizing the vitality of behavioural finance to practical finance. The remainder of this paper will discuss as follows: The second section discusses investment and behavioural finance literatures. The third section analyses data and methodology used in this paper. Fourth section presents empirical results. The last section is the conclusion.

2. Literature Review

Markowitz (1959) asserts that investors will not pay a premium to the idiosyncratic risk because it can be eliminated by diversification. Thus, they only pay for market risk as a reward to the risk. Eiling et al*.* (2012) confirm that diversification across countries benefits investors as a result of the difference between the correlations among countries. Driessen (2007) asserts that international diversification values investors significantly, especially investors from less developed countries. Similarly, Americans would be advantageous from international diversification, if they invest outside a domestic market. Despite the advantage of diversification, which help investors to reduce idiosyncratic risk, numerous studies show that investors have experienced 'home-bias' (Chiou 2009).

Lewis (1999) measures 'home-bias' and concludes that developing countries such as Indonesia, Thailand, and Egypt have higher home-bias ratio than developed countries. Furthermore, Huberman (2001) finds that Germans perceive domestic stocks more competent because of familiarity. This study is later supported by Boyle *et al.*, (2012) which conclude that level of familiarity leads to high participation in a stock market. An alternative behavioural justification is that investors might feel more familiar with domestic market because they can understand language and culture. Hence, they can access to the information, resulting in their subjective distribution of returns to have low dispersion (Kilka & Weber 2000). Epstein & Schneider (2008) confirm the previous one by introducing model which demonstrates the effects of ambiguity information. They conclude that ambiguity leads to excess volatility, equity premium puzzle and negative skew of returns. As a result of this, the expected utility theory is inappropriate to explain an uncertainty situation.

Considering Ellsberg's paradox as an example, regardless of risk aversion, an individual would prefer a choice which one knows the chance of winning to a gamble on unknown (Ellsberg 1961). As illustrated in Gilboa & Schmeidler (1989), Qu (2013) and Schmeidler (1983), they develop a new model such as Choquet expected utility and Max-min expected utility to represent behaviour under an uncertainty situation. As for Max-min expected utility, a decision maker will maximize the minimal expected utility with respect to a set of priors. This case is so called multiple priors model which Garlappi *et al.* (2007) apply to incorporate ambiguity aversion. They suggest that multi-prior approach differs from Bayesian approach in several ways. First, optimal portfolio incorporates with an ambiguity-averse investor rather than an ambiguity-neutral investor. Second, the unknown parameters are treated by the degree of uncertainty which allows decision maker to choose their level of ambiguity, not as a random variable. Not surprisingly, they find that optimal portfolio from multi-prior method is less unbalanced and has higher Sharpe ratio than traditional mean-variance portfolio. However, in reality, the distribution is uncertain in both mean and variance. Therefore, this paper extends the scope of aforementioned research by studying the relationship between ellipsoidal uncertainty level and the optimal weight, in an uncertainty situation in both mean and variance, without overlooking the risk aversion coefficient of individual preference.

3. Data and Methodology

3.1 Data

To test the case with three risky assets, this research employs weekly return data from three different stock indexes which have sufficiently different characteristics: the number of participants, trading volumes, and sizes of firms. Hence, NYSECOMPOSITE and BANGKOK S.E.T are selected in this research. S&P 500 COMPOSITE is added to the case of three risky assets. Weekly returns from 13 May 1975 to 2 July 2013 were obtained from DataStream 5.0. A sample in each market consists of 1,991 observations over a 29-year period.

3.1.1 Descriptive Statistics

Addition to the Lilliefors test, it can be seen that three markets cannot be accepted the null hypothesis that they have the normal distribution returns at 5% significant level. Panel B depicts Variance-Covariance matrix where the diagonal of the matrix denotes the variance of each market. The rest is the covariance of three variables.

3.1.2 Hypothesis Testing for Lilliefors Test

$H_0: return\ has\ normal\ distribution$
$H_1: return\ has\ non-normal\ distribution$

Table 1: Descripive statistics of weekly returns

Panel A – S&P500 COMPOSITE, NYSE COMPOSITE, and BANGKOK S.E.T. from 13 May 1975 to 2 July 2013

	Mean (%)	Standard deviation (%)	Hypothesis testing	P-VALUE	KSTAT
S&P 500 COMPOSITE	0.1726574	0.278236	1	0.001	0.043625
NYSE COMPOSITE	0.1724965	0.2941641	1	0.001	0.051954
BANGKOK S.E.T.	0.2026988	0.2086791	1	0.001	0.071361

Panel B - Variance-Covariance matrix represented in %

Variance-Covariance matrix	S&P 500 COMPOSITE	NYSE COMPOSITE	BANGKOK S.E.T.
S&P 500 COMPOSITE	0.0529	0.0512	0.0204
NYSE COMPOSITE	0.0512	0.0513	0.0217
BANGKOK S.E.T.	0.0204	0.0217	0.136

3.2 Methodology

Following Garlappi et al. (2007) and Boyle et al. (2012), this paper will develop their model to test the change of optimal weight portfolios with ambiguity level. The paper starts with the classical mean-variance model, and then introduces a multi-prior model. Finally, it closes with a new model which is a model under uncertainty in mean and variance.

3.2.1 Proposition 1: Classical Mean-Variance Portfolio

To start, the classical mean-variance model proposed by Markowitz (1959) is used to obtain the optimal portfolio weight.

$$\max_w w'\mu - \frac{\Upsilon}{2} w'\Sigma w \qquad (1)$$

Subject to

$$w'\mu = \mu_p$$
$$w'1 = 1$$

3.2.2 Proposition 2: Uncertainty in Variance

The study of Garlappi et al. (2007) is only concerned with uncertainty in mean. This work introduces a new model of risky assets with uncertainty in variance. Barry (1974) mentions in his paper that the supposed mean and covariance matrix are unknown; a marginal distribution of covariance matrix is normal-inverted Wishart. However, this paper focuses on variance, and will utilize chi-square distribution to explain probability distribution. First of all, when variance is unknown, using chi-square distribution to describe variance distribution is essential.

$$(n-1)\frac{\hat{\sigma}^2}{\sigma^2} \sim \chi^2 \qquad (2)$$

Second, confidence intervals are relative to ambiguity level. Similarly, an intuitive way to find upper and lower bounds of variance distribution is to find the confidence interval of variance.

$$\frac{\hat{\sigma}^2(n-1)}{\chi^2_{\frac{\alpha}{2}}} \leq \sigma^2 \leq \frac{\hat{\sigma}^2(n-1)}{\chi^2_{1-\frac{\alpha}{2}}} \qquad (3)$$

Based on Gilboa and Schmeidler (1989), investors update their beliefs by maximizing the worst case situation which means, in this case, investors will choose the upper bound of variance for long position to process the utility, and the lower bound of variance for short position.

$$\max_{\pi} \min_{\sigma} w'\mu - \frac{\Upsilon}{2} w'\Sigma w \qquad (4)$$

$$w'\Sigma w = w^2\sigma_1^2 + 2w(1-w)\sigma_{12} + (1-w)\sigma_2^2 \qquad (5)$$

The worst case variance is selected either from upper bound or lower bound.

$$\sigma^2{}_{adj} = \begin{cases} \dfrac{\hat{\sigma}^2(n-1)}{\chi^2_{\frac{\alpha}{2}}}, & \pi_n < 0; short\ position \\ \dfrac{\hat{\sigma}^2(n-1)}{\chi^2_{1-\frac{\alpha}{2}}}, & \pi_n > 0; long\ position \end{cases}$$

$$\sigma_n^2 = \sigma_{adj}^2 \quad ; \text{n} = 1, 2 \qquad (6)$$

3.2.3 Proposition 3: Uncertainty in Mean and Variance

In the preceding model, the mean is fixed, while variance is varied. This proposition suggests a new model in which both mean and variance are uncertain. According to Barry (1974), in the case of mean and variance being unknown, mean will be conditional upon covariance matrix. Nevertheless, this study represents mean and variance in a different model.

To start with Sharpe ratio

$$\mu' = r + \lambda\sigma \qquad (7)$$

In this situation, return will simply vary by variance. Interestingly, variance which is selected for long position may not be an upper bound as in proposition 3. Similarly, short position may not have variance from the lowest bound. One has to balance between increase in variance and increase in return, and select the variance which minimizes the inner maximization.

Objective function: $f(w) = w'\mu - \frac{\Upsilon}{2} w'\Sigma w$ (8)

Constraint: $w'\mu = \mu_p$

$W'1 = 1$

$$\frac{\hat{\sigma}^2(n-1)}{\chi^2_{\frac{\alpha}{2}}} \leq \sigma^2 \leq \frac{\hat{\sigma}^2(n-1)}{\chi^2_{1-\frac{\alpha}{2}}}$$

To find the closed form equation, one can use the Lagrangian method by taking the partial derivatives of the Lagrangian objective function with respect to the $w_1, w_2, \ldots, w_n, \lambda_1, \lambda_2$ variables. Then, the weights from this differentiation are optimal weights for this model. In this circumstance, there is no short-selling constraint; however, in fact, several funds are permitted from short-selling. Hence, if fund managers want to add a short-sale restriction in the model, they have to add the constraint.

$$w_n \geq 0 \; ; \; n = 1, \ldots, N$$

4. Result

The results are broken down into three parts. The first section is a main finding of the relationship between the optimal weights and level of ambiguity based on two models: (I) the mean-variance portfolio with uncertainty in variance (II) the mean-variance portfolio with uncertainty in mean and variance. The second part explains the effect of risk aversion to optimal weights in two models. The third section scrutinizes whether the uncertainty model performs better than classical model.

4.1 Three Risky Assets

Three stock markets are S&P500 COMPOSITE, NYSE COMPOSITE and BANGKOK S.E.T. In this paper first asset refers to S&P 500, second asset is NYSE, and third asset is S.E.T.

market. Following (Markowitz 1959) approach, the optimal weights are shown in Table 2 with constraints of full investment and no short-selling restriction.

Table 2: The optimal weight from classical portfolio

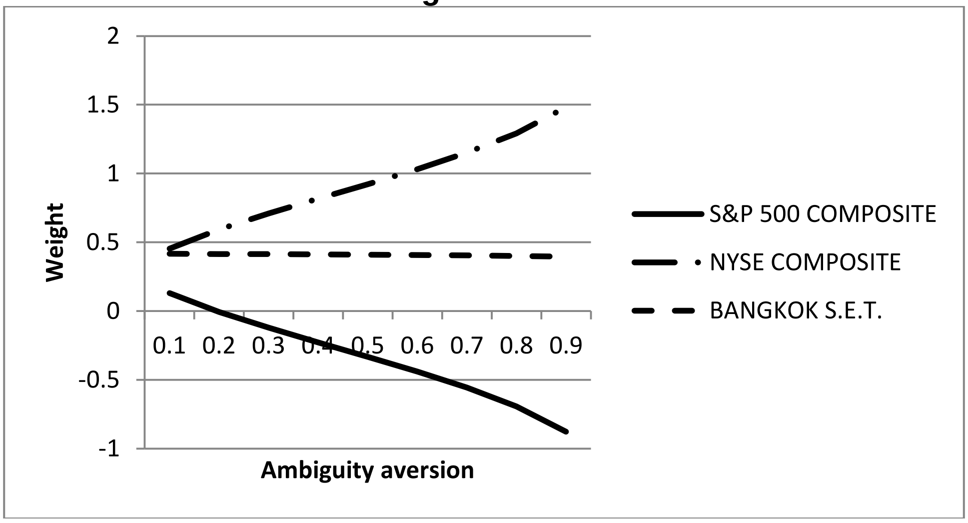
Optimal	W
S&P 500 COMPOSITE	0.440712
NYSE COMPOSITE	0.14013
BANGKOK S.E.T.	0.419159

Table 3: Variance-Covariance matrix of three markets represented in % calculated from weekly returns.

Variance-Covariance matrix	S&P 500 COMPOSITE	NYSE COMPOSITE	BANGKOK S.E.T.
S&P 500 COMPOSITE	0.0529	0.0512	0.0204
NYSE COMPOSITE	0.0512	0.0513	0.0217
BANGKOK S.E.T.	0.0204	0.0217	0.136

4.1.1 First Model: Uncertainty in Variance

Figure 1: The optimal weight for the first model when only S&P500 COMPOSITE is ambiguous.



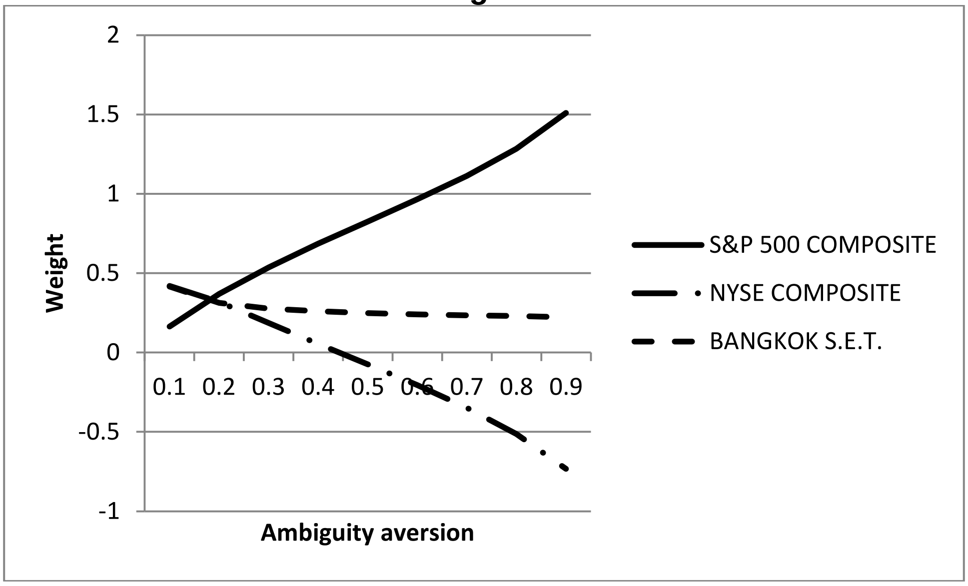
With respect to Figure 1, when the S&P 500 is unfamiliar, the optimal weight will reverse significantly as shown in Figure 1. At ambiguity level 0.1, NYSE COMPOSITE which has the lowest weight in the classical mean-variance model turns to be the most attractive at 0.42. This is because S&P 500 and NYSE have a similar structure, however, when investors are unsure about S&P 500, they tend to switch their spending into another similar market. The weight in S&P 500 decreases sharply from 0.17 to approximately -0.7, after increasing ambiguity level from 0.1 to 0.9. Meanwhile, BANGKOK S.E.T remains stable over time.

Consequently, the difference in each position expands with the level of ambiguity. At ambiguity level 0.1, NYSE has roughly 0.3 greater proportion of wealth than S&P 500, however, at level 0.9, NYSE significantly dominates S&P 500 with more than 2.4 proportion.

4.1.2 Second model: uncertainty in return and variance

Ambiguity in first asset

Figure 2: The optimal weight for the second model when only S&P500 COMPOSITE is ambiguous.



Interestingly, in an uncertainty in mean and variance world, the result is contrary to the first model. Instead of avoiding unfamiliar asset like in the first model, investors are more willing to invest in an unfamiliar asset. The reason for this is that mean returns are fixed in the first model. Hence, investors will maximize the worst case by choosing the upper bound of a variance to ensure that they will minimize the utility in each situation. Notwithstanding, in the second model, returns are dependent on the product of Sharpe ratio and variance. Selecting the upper bound does not lead to minimizing utility owing to increase in returns. It can be seen that the variance of the second model has a downward trend with ambiguity level in the second model.

To conclude, unknown return distribution leads investors to avoid an unfamiliar asset, whereas, unknown in mean and variance parameter increase attention to risky assets.

Table 4: The variance of the second model when only S&P500 COMPOSITE is ambiguous.

First model: the mean-variance model with uncertainty in variance at different level of uncertainty.

Second model: the mean-variance model with uncertainty in mean and variance at different level of uncertainty.

One ambiguity	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
First model	0.0233	0.0234	0.0236	0.0237	0.0238	0.0239	0.0240	0.0242	0.0245
	0.0226	0.0226	0.0226	0.0226	0.0226	0.0226	0.0226	0.0226	0.0226
	0.0369	0.0369	0.0369	0.0369	0.0369	0.0369	0.0369	0.0369	0.0369
Second model	0.0227	0.0226	0.0225	0.0224	0.0222	0.0221	0.0220	0.0219	0.0216
	0.0226	0.0226	0.0226	0.0226	0.0226	0.0226	0.0226	0.0226	0.0226
	0.0369	0.0369	0.0369	0.0369	0.0369	0.0369	0.0369	0.0369	0.0369

4.2 Risk Aversion

In reality, individual has a different level of risk tolerance. More risk-averse investors will prefer holding the risk-free asset to the risky asset. Lei *et al.* (2009) demonstrate that the risk aversion level affects the diversification level positively. To capture the effect of risk aversion in the uncertainty model, this section tests the relationship between risk aversion and the optimal weight, and when only first asset is ambiguous.

4.2.1 First Asset Risk Aversion

First model: uncertainty about variance

Figure 3: The relation between risk aversion and the optimal weight of S&P 500 under the first model portfolio when only S&P500 COMPOSITE **is ambiguous**

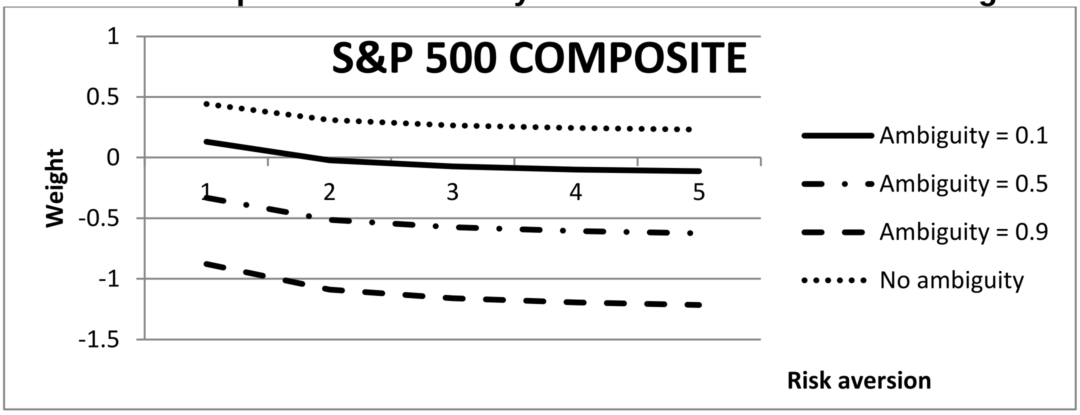


Figure 4: The relation between risk aversion and the optimal weight of NYSE under the first model portfolio when only S&P500 COMPOSITE is ambiguous.

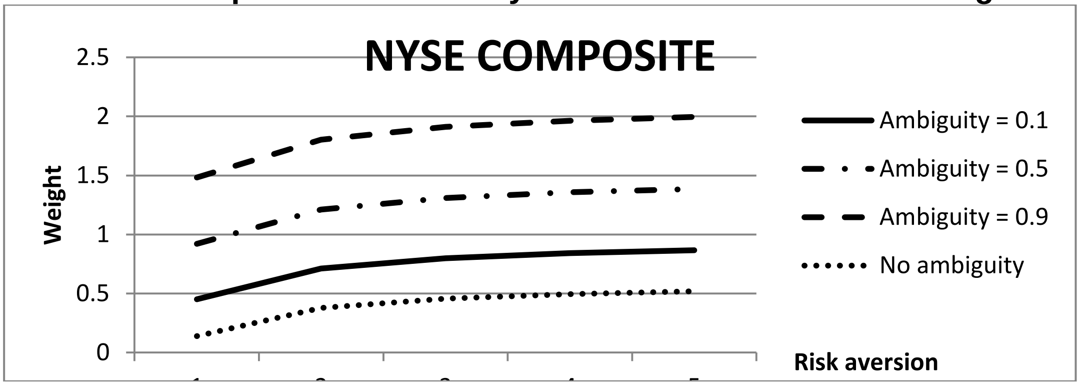
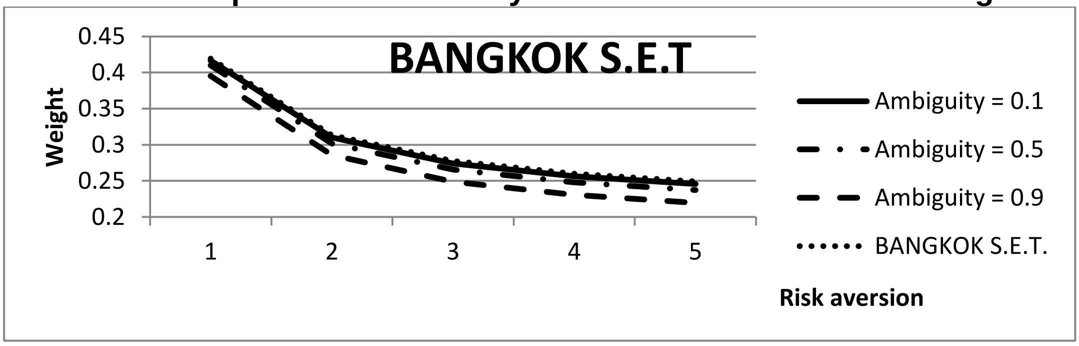


Figure 5: The relation between risk aversion and the optimal weight of S.E.T. under the first model portfolio when only S&P500 COMPOSITE is ambiguous.



In this part, only S&P 500 COMPOSITE has an uncertainty in variance. Investors tend to devote less in an unfamiliar asset; the position in S&P 500 declines drastically with increase in risk aversion. From the classical optimal portfolio, NYSE is an unattractive market, however, when S&P 500 is treated as an unfamiliar asset, NYSE COMPOSITE is more demanded. Therefore, it can be concluded that S&P 500 COMPOSITE and NYSE COMPOSITE are nearly perfect substitutes. If one is decreased in the proportion, the other will increase.

Second model

Figure 6: The relation between risk aversion and the optimal weight of S&P 500 under the second model portfolio when only S&P 500 is ambiguous.

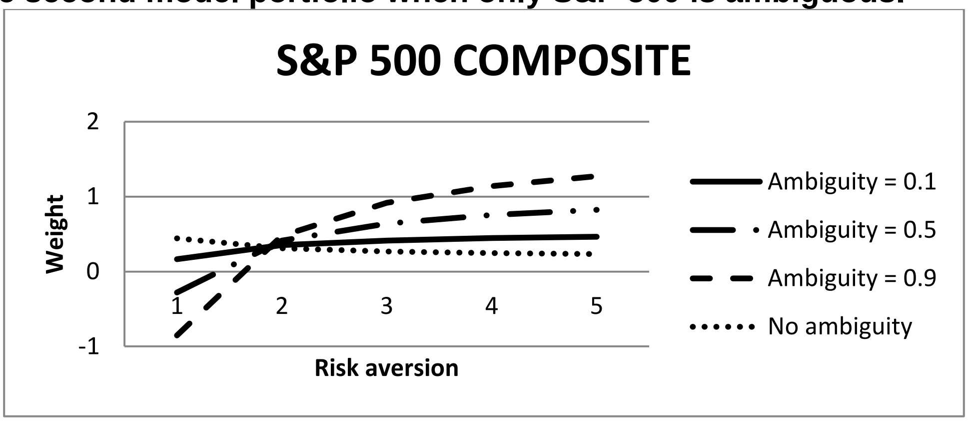


Figure 7: The relation between risk aversion and the optimal weight of NYSE under the second model portfolio when only S&P 500 is ambiguous.

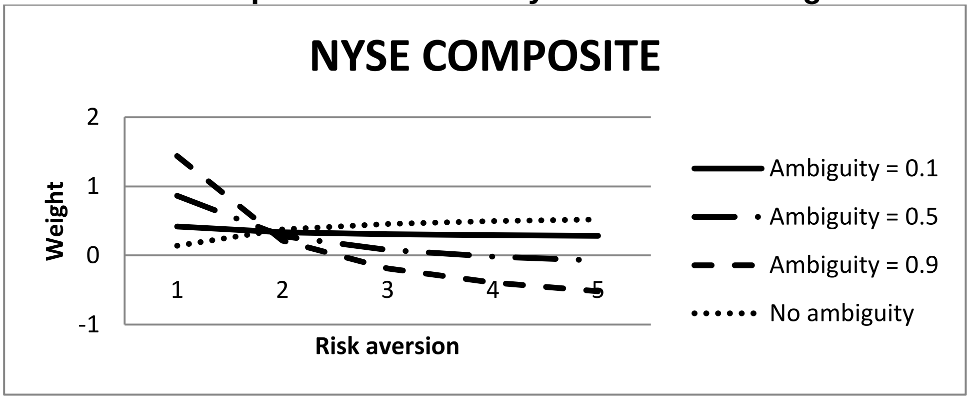
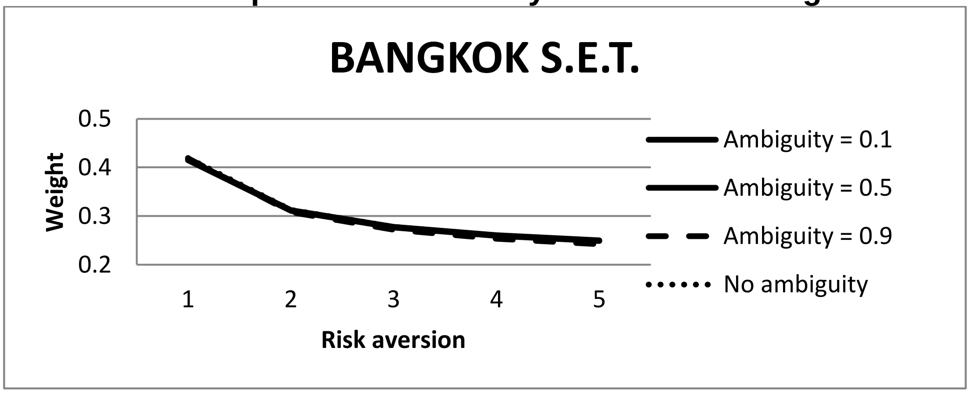


Figure 8: The relation between risk aversion and the optimal weight of S.E.T. under the second model portfolio when only S&P 500 is ambiguous.



As Kogan and Uppal (2000) suggested that the optimal weights can rise or decline with the risk aversion level. Started from an unfamiliar S&P 500 market uncertainty level at 0.1, investors spend less in the second model than in the classical model at 0.16 and 0.44 respectively. However, the position has changed when risk aversion goes up above 2; investors take more risk in an unfamiliar asset. As for NYSE market, even though it is familiar asset, it is less attractive in terms of returns and risk. Risk-averse investors feel less favour at risk aversion higher than 2. Contrastingly, investors consistently invest in S.E.T regardless of ambiguity level; the position has negative correlation with risk aversion.

4.3 Performance

Numerous empirical findings suggest that the classical mean-variance portfolio performs poorly compared to factor portfolios (Demiguel et al. 2009). This section analyzes the performance of the classical model, relative to the performance of equal weight, the first model, and the second model portfolios.

The result is shown in Table 6 panel A; the classical model outperforms the first model with 70-week estimation period and with 120-week estimation period. The exception is on the 0.1 ambiguity first model with 120-week estimation period, where it has higher Sharpe ratio. Moreover, only the 0.9uncertainty level portfolio with 120-week estimation period has confirmed Sharpe ratio at a significant level of 10%.

Table 5: Mean of Sharpe ratio of out-of-sample portfolios calculated from weekly returns between 13 May 1975 and 2 July 2013.

Panel A. Comparison between the classical portfolio performance and the portfolio with ambiguous in first assets

Classical model: $\mathbf{max}_w\, \mathbf{w}'\mu - \frac{\gamma}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}$

First model: the mean-variance model with uncertainty in variance at level 0.1, 0.5, 0.9 respectively.

Second model: the mean-variance model with uncertainty in mean and variance at level 0.1, 0.5, 0.9 respectively.

Portfolios are estimated on a rolling basis from the last 70 weeks and 120 weeks.
T-statistic is represented in bracket and * indicates statistical significance at 10% level.

		Uncertainty level					
		0.1		0.5		0.9	
Week	Classical portfolio	First model	Second model	First model	Second model	First model	Second model
70	0.0795	0.0785	0.0790	0.0712	0.0757	0.0612	0.0603
		(0.4562)	**(0.1528)**	**(0.8356)**	**(0.2506)**	**(1.1895)**	**(0.9401)**
120	0.0896	0.0898	0.0877	0.0868	0.0812	0.0676	0.0670
		(-0.0798)	**(0.3636)**	**(0.2482)**	**(0.4792)**	**(1.3759)***	**(1.0414)**

Considering the classical portfolio, it has by far the highest variance among other portfolios. However, it can be seen that mean excess return of the classical one outperforms uncertainty portfolios with average 2.58% and 1.77% weekly returns. Therefore, investors are more willing to take more risk because the return rewards to the risk.

Table 6: Mean excess weekly return and variance of out-of-sample portfolios calculated between 13 May 1975 and 2 July 2013.

Panel A: Comparison between mean excess returns from the classical portfolio performance and that from the portfolio with ambiguity in S&P 500.

		Uncertainty level					
		First 0.1		First 0.5		First 0.9	
Week	Classical portfolio	First model	Second model	First model	Second model	First model	Second model
70	0.0258	0.0230	0.0232	0.0172	0.0155	0.0168	0.0175
120	0.0177	0.0166	0.0168	0.0088	0.0109	0.0083	0.0110

Panel B: Comparison between variance from the classical portfolio performance and that from the portfolio with ambiguity in S&P 500.

		Uncertainty level					
		First 0.1		First 0.5		First 0.9	
Week	Classical portfolio	First model	Second model	First model	Second model	First model	Second model
70	0.1997	0.1813	0.1860	0.1576	0.1674	0.1654	0.1838
120	0.1657	0.1541	0.1570	0.1147	0.1349	0.1292	0.1394

Overall, it can be concluded that the classical model is still applicable nowadays as we can see that its-out-of-sample Sharpe ratio performance is higher than the both uncertainty models. The results contradicts Garlappi et al*.* (2007) where the research suggests that uncertainty model outperforms the classical model.

5. Conclusion

This paper has examined the impact of uncertainty in mean-variance optimal portfolio, and its out-of-sample performance. In general, uncertainty in parameters is ignored in Markowitz's portfolio model. However, we have shown how uncertainty in parameters such as variance-covariance matrix affects the investment decision. Ambiguity aversion in variance is added in constraints on the mean-variance model. We considered two models, namely: uncertainty in variance and uncertainty in mean and variance models. Both models have significantly been different in each optimal weight. The empirical pieces of evidence demonstrate that under an uncertainty in variance model, investors avoid holding an unfamiliar asset. Hence, one potential explanation for the home-bias puzzle is that investors are confident about the variance distribution of domestic asset resulting in putting more weight in that asset. Investors feel more advantage when they know return distribution, especially when an ambiguity rises. Discussed in Brennan et al. (2005), the disadvantage in information creates an uncertainty in the foreign market. Nevertheless, under an uncertainty in mean and variance model, shareholders prefer to take risk in an ambiguous asset. The reason for the difference is that we assume that return is known in the first model, while returns in the latter model are varied with the product of Sharpe ratio and variance.

Employing the classical mean-variance model, investors restrict several assumptions such as known return distribution. However, one cannot reject that the-out-of-sample Sharpe ratio performance of this classical model performs worse than the portfolio with uncertainty model.

However, we argue that it is inappropriate to use this classical model in reality. To explain, weekly returns of the two models outperform that of the classical model, particularly portfolios from the 120-week estimation period. Interestingly, even though portfolio weights from the second model are extreme in some situations such as portfolios in which only S&P 500 is ambiguous, the performance from those portfolios outperforms significantly that from the classical portfolio, although we could not conclude that the portfolio with uncertainty models is less fluctuated than the classical one.

To further develop the optimal model with uncertainty, one could consider time variation in Sharpe ratio. Investors might acquire an appropriate optimal model in practice, if Sharpe ratio changes over time.

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