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Project Scheduling Problem, Quality Loss and Time-Cost Tradeoff

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The competitive market compels the project owners for ensuring higher performance standards, faster delivery and reductions in costs. It is a great challenge for project-based organizations to satisfy these requirements. Crashing project activities is a process of accelerating a project to meet due date. The project quality may be affected by excessive project crashing. In this paper, a modified mixed integer linear programming technique is considered taking into account the potential quality loss cost and with direct and indirect costs for excessive crashing activities. The modified mathematical model is applied to a real-life scenario. Then a comparison between existing and modified model is shown.

Keywords: Project scheduling, potential quality loss cost, time-cost-quality tradeoff, direct and indirect cost, mixed integer linear programming etc.

1. Introduction

Completing projects on scheduled time is an indicator of the efficiency of the project. But projects are subject to many variables and unpredictable factors, which result from many sources causing the delay of the project. There may be significant variance between the assumptions made regarding a project and actual outcomes. Crashing project activities is generally accepted by the project managers to shorten the completion times to meet project due dates. Again, sudden unexpected changes in construction technology, techniques, materials, or human resources can create scheduling pressures that in turn may increase the possibility of failure of a project. Also, the excessive crashing of project activities causes quality degradation of the project. In traditional project scheduling problems, only the time and cost are considered without the quality parameters. But quality is one of the main pillars of a successful project. That is why quality should be effectively considered in managing of industrial projects. A project is a one-time task constrained by time, cost, and quality. The project's success depends on how well these constraints are balanced. If any of the constraints is over-emphasized, burdens may fall on the other two. Hence, crashing project activities should be considered a significant factor in the time–cost tradeoff problem. In this paper, the quality parameter is incorporated with the traditional time–cost trade-off problem to develop a time, cost, and quality trade-off problem (TCQTP) with some practical assumptions.

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A modified mathematical model taking into account the potential quality loss cost (PQLC) and direct and indirect cost for crashing activities has been proposed. The main objective of this research work is to modifying a mathematical model to visualizing total crashing cost. This model includes quality loss cost for excessive crashing activity and also the indirect cost for crashing activities of a project. Adding indirect cost in the calculation of total cost is the uniqueness of this paper. Another objective is to apply the modified mathematical model to a real-life problem to test its feasibility.

In industrial and manufacturing area, a number of problems have been solved in the time cost trade-off problems but most of them have been proposed without the consideration of activity quality. Hence, the existing model without considering the quality of activity is too optimistic (Kim, Kang & Hwang 2012). To determine and predict the total crashing cost of each activity of a project, a mixed integer linear programming (MILP) model is proposed. The optimal total crashing cost will be determined considering the quality of the project. In this paper focused on optimization of total crashing cost of a production belt of lamp production. The minimal extra cost of crashing of production belt is also focused in this modified model. A project scheduling problem considering total costs (including direct & indirect cost) when crashing activity occurs in a project to minimize the completion time Kim, Kang and Hwang (2012) mentioned the total cost considering PQLC are minimized under some following assumptions

- Precedence relationship should be maintained in the case of start an activity
- The direct crashing cost bound the rework or modification cost
- Required unlimited resource
- Non linearity should not be considered
- Non-conformance activity requires rework or modification

Recent researchers have solved MILP to compute direct project costs, considering non-conformance risk activity rate and PQLC of crashing project activities. But they didn’t consider indirect cost of the crashing project. In this paper, a mixed integer linear programming model has been proposed considering the total crashing cost (including direct & indirect cost) that accounts for both the non-conformance risk rate and PQLC of the project crashing activities. In the modified model cost is reduced by the existing model.

The paper is organized as follows: section 2 deals with the literature review; section 3 mentions the research methodology, section 4 represents problem statement, section 5 represents mathematical statement, section 6 focuses on data collection and analysis, section 7 represents results analysis, section 8 represents Discussion, section 9 represents conclusion, section 10 represents limitations. Finally, the references and appendix are mentioned in the last portion of this paper.

2. Literature Review

Time, cost and performance (outcomes or deliverables) are the three main dimensions in project management through which projects may be featured. Therefore, a project must be managed in order to optimize these three dimensions. Time and cost must be minimized, but performance must be maximized. The three dimensions are highly correlated. Time management in project management is vital. From the last decade or so, CPM and PERT were the fundamental techniques to solve project scheduling problems that minimize the project completion time, while satisfying the precedence relationships among the activities, with an assumption that the resources are unlimited.

The critical path method (CPM) that is used for all types of projects, such as construction, engineering, facility maintenance, software development, and research and development is a mathematical algorithm used to schedule a set of activities in a project. This method is fundamentally related to the trade-off between completion time and the costs of the project. In the late 1950s, a number of problems have been solved in the time cost trade-off problems. The problems have been proposed by linear programming (LP), non-linear programming (NP), integer programming (IP) and mixed integer linear programming (MILP) and heuristic algorithms (HA). But most of the models have been proposed without the consideration of activity quality. Although, in the late 1990s, some problem associated with activity quality have been proposed in the time cost trade-off problem. But most of the problems were not applicable for real-life project. The quality of activity should be taken into consideration in a crashing project.

An interrelationship among time, cost & quality associated in a linear programming model were proposed in a study of Babu and Suresh (1996). They adopt the continuous scale to determine each activity quality. The different ways of function e.g. arithmetic mean geometric mean attained the project quality which is assumed by different quality levels. But this continuous scale at the activity level is inapplicable for solving real problems. Sakellaropoulos and Chassiakos (2004) solved a mixed integer linear programming model. Considering some parameters e.g. generalized precedence relationships, activity planning constraints, external activity constraints, and late penalty/early bonus existence, the model provides an optimal project time-cost curve and a minimum cost schedule without consideration of the quality of project activities. Tareghian and Taheri (2007) solved a problem involves the scheduling of project activities in order to minimize the total cost of the project while maximizing the quality of the project and also meeting a given deadline.

The problem-based on meta–heuristic solution procedure and solves discrete time, cost, and quality tradeoff problems But the maximum quality of is unassisted. Hence this problem may be also inapplicable for real project. Cohen, Golany and Shtub (2007) solved the TCTP is typically stated either as a minimization of the project’s costs under a specified due date or as a minimization of the project’s duration under a given budget. The model considered the problem of allocating resources to projects performed under given due dates and stochastic time–cost tradeoff. After that, Vanhoucke and Debels (2007) developed a new meta-heuristic procedure to provide near-optimal heuristic solutions for different problems. But the model needs to improve this procedure for the discrete time/cost tradeoff problem with times-witch constraints or net present value maximization. They also have no consideration for quality of project activities. Wuliang and Chengen (2009) proposed an optimization model consist of multi-mode resource constrained time cost trade off (MRCTCT) problem solved by genetic algorithm (GA). The proposed model has reduced the total project cost with respect to the resource restrictions. The suggested GA technique in this research has higher flexibility to solve problems compared to other methods.

A mixed integer linear programming model was proposed by Kim, Kang and Hwang (2012) and procedure that takes into account the direct crashing cost and potential quality loss cost in the time-cost trade-off problem. The PQLC associated with rework or modification cost in case of crashing activities also provided in this model. Under some assumptions e.g. generalized precedence relationship bounded rework or modification with crash time, bounded rework or modification cost with crashing direct cost, the model has been solved. In this paper, MILP has been used to compute direct project costs, considering non-conformance risk activity rate and PQLC of project activities. This procedure makes it applicable to solve any kind of real project problem.

Throughout the literature review, it is seen that a number of problems have been solved in the time cost trade-off problems. Hence, existing models and procedures without the consideration of the quality of activity are too optimistic. But considering quality, the original cost cannot be optimized. Many researchers pointed out the requirement of analyzing all elements of quality costs to make optimal decisions. But a few had consulted about crashing cost of a crashing project.

Some papers have computed direct project costs, considering non-conformance risk activity rate and PQLC of crashing project activities. But they didn't consider indirect cost of the crashing project. In this paper, a mixed integer linear programming model has been proposed considering the total crashing cost (including direct & indirect cost) that accounts for both the non-conformance risk rate and PQLC of the project crashing activities.

3. Research Methodology

Some relevant papers on time-cost tradeoff project scheduling problem have been studied. A relevant paper has been chosen to minimize cost for crashing projects. Quality has been given importance in these studies. In the selected paper the additional cost i.e. indirect cost is included for the better understanding of the total cost for crashing activities. A mixed integer linear programming (MILP) model is modified to determine the optimal total crashing cost considering PQLC. The modified mathematical model has been solved by LINGO 13.0 with branch and bound algorithm of mixed linear integer programming. To prove the feasibility of the proposed models, it is applied to a real-life problem. The methodology of this research work is shown in a flow chart. The details procedure is visualized in a brief.

4. Methodology

4.1 Existing Mathematical Model Formulation

Variables,

Y_j = Reduced crash time for the completion of activity j
Z_j = 1 if activity j is selected as a nonconformance risk activity, otherwise 0

Parameters,

m_j = Direct cost per unit time for activity j
t_j = Normal time required when activity j is performed under normal conditions
c_j = Normal direct cost when activity j is performed in the normal time tj
t'_j = Crash time required to complete activity j by assigning resources beyond those originally allocated
c'_j= Crash direct cost when activity j is completed in the crash time t′j

E = Additional direct cost when activity j is completed in crash time t′j
R_j = Allowable reduced time for activity j
X_j = Start time for activity j
X_i = Start time for predecessor activity i
K = Normal completion time for predecessor activity i
Y_i= Crash time for predecessor activity i
X_n= Start time for activity n
t_n= Normal completion time for activity n
Y_n= Crash time for activity n
D = Due date of the project,
N = Number of activities in the project
q_j= Potential quality loss cost for activity j
α = Nonconformance risk activity rate predetermined by the project manager
k = an arbitrarily large number

The model formulation,

Min $\sum_{j=1}^{n} \mathrm{mjYj} + \sum_{j=1}^{n} \mathrm{qjZj}$
Subject to
$Y_j \le R_j$ for all j ………………………………………………….......... (1)
$X_j \ge X_i + (K - Y_i)$ ……………………………………………………… (2)
$X_n + t_n - Y_n \le D$ ……………………………………………………… (3)
$\Sigma Z_j \le N * \alpha$ ……………………………………………………………… (4)
$kY_j \ge Z_j$ ………………………………………………………………… (5)
$Z_j \in \{1; 0\}$…………………………………………………................ (6)
$Y_j \ge 0$…………………………………………………………………….. (7)
$m_j = (c'_j - c_j)/(t_j - t'_j)$ ………………………………………………….. (8)

The objective function defines the direct crashing cost considering potential quality loss costs where rework or modification cost required. The above formulation includes seven constraints.

In the first constraint (1) the time for each activity cannot be reduced by more than its maximum time reduction. Second constraint (2) states that the start time of each activity must be at least as great as the finish time of all of the immediate predecessors because the activity finish time is reduced by the amount of time that each activity is crashed. The third constraint (3) indicates that the project must be completed by its due date. The fourth constraint (4) states that the project manager limits the number of nonconformance risk activities in the project. In fifth constraint (5) the arbitrarily large number k is given to prevent the equation from becoming binding. In equation (6), Z_j defines 1 if activity j is selected as a nonconformance risk activity, otherwise 0. Y_j in equation (7) defines that it will always be greater or equal than 0. Equation (8) represents the incremental direct crashing cost per unit time for each individual activity.

In this existing mathematical model, there are few limitations. Project costs are generally classified into two categories: the direct costs related to individual activities of the project and the indirect costs related to overhead items. The direct costs, considering the PQLC are minimized but indirect costs haven’t measured in the existing model. The following mixed integer linear programming model considering the direct and indirect cost for crashing activities including PQLC is proposed.

4.2 Modified Mathematical Model Formulation

The parameters of existing model will be followed in the modified model. Some additional parameters for the modified model are given below:
F = Additional indirect cost when activity j is completed in crash time t′j
b_J= Indirect cost per unit time for activity
ic = Normal indirect cost when activity j is performed in the normal time tj
ic'_j=Crash indirect cost when activity j is completed in the crash time t′j

The model formulation,

Min $\sum_{j=1}^{n} mjYj + \sum_{j=1}^{n} qjZj - \sum_{j=1}^{n} bjYj$
Subject to
$Y_j \leq R_j$ for all j ………………………………………………… (1)
$X_j \geq X_i + (K - Y_i)$ ………………………………………………… (2)
$X_n + t_n - Y_n \leq D$ ………………………………………………… (3)
$\Sigma Z_j \leq N * \alpha$ ………………………………………………… (4)
$kY_j \geq Z_j$ ………………………………………………… (5)
$Z_j \in \{1; 0\}$………………………………………………… (6)
$Y_j \geq 0$ ………………………………………………… (7)
$m_j = (c'_j - c_j)/(t_j - t'_j)$ ………………………………………………… (8)
$b_j = (ic_j - ic'_{j\ j})/(t_j - t'_j)$ ………………………………………………… (9)

The objective function incorporates the direct and indirect crashing costs considering PQLC. Up to 8th constraints, the meaning remains same as existing model. Equation (9) represents the indirect crashing cost per unit time for each individual activity.

5. Data Collection and Analysis

It is known that mixed integer linear programming (MILP) involves problems in which only some of the variables, are constrained to be integers, while other variables are allowed to be non-integers. Zero-one linear programming involves problems in which the variables are restricted to be either 0 or 1. Mixed integer programming has many applications in industrial production.

A possible objective is to maximize the total production, without exceeding the available resources. One of the most important issues in the application of mixed-integer programming techniques to the process scheduling problems lies in the computational efficiency for the solution of the resulting MILP problems. The MILP based scheduling approaches have been employed to a wide variety of real-world problems (Floudas & Lin 2005).

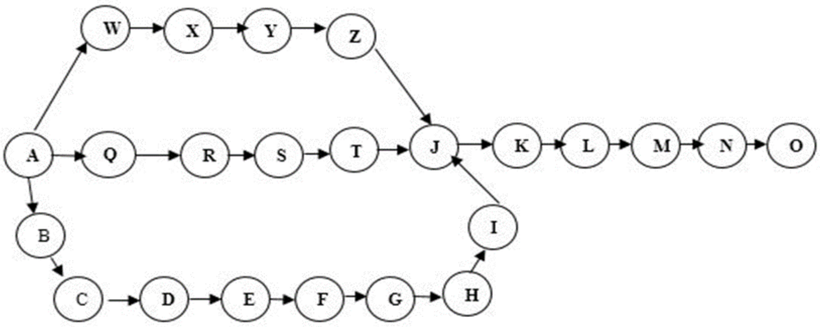
In this section, proposed models have been estimated in real life project. To perform the validity test, a prominent company for lamp production has been chosen. The factory is situated in Dhaka, Bangladesh. Now there are three types of bulbs of each brand are produced in the factory of this company. They are GLS (General lighting service), CFL (Compact Fluorescent Lamp), FTL (Fluorescent tube Light).

GLS production belt consists of a semi-automation program. The speed of the belt is 3000 piece bulbs per hour. 17 personnel need to operate the machine. 40W, 60W, 100W, 200W bulbs are produced by the machine. They have the chance to unfixed production rate and change the speed of production per hour. These types of data would satisfy the requirement of the proposed model.

5.1 Critical Path Determination

In this project, there are 23 activities. They are A) Flare Making, B) Stem Making, C) Annealing Oven, D) Mount Making, E) Marking & Shell, F) Sealing, G) Pumping, H) Cap Filler, I) Cap Rail, J) Capping Mill, K) Flashing, L) Cap Soldering, M) Repairing, N) Pick Up, O) Glow Test, Q) Stem Machine Installation, R) Mount Machine Installation, S) Sealing Machine Installation, T) Capping Mill Installation, W) Wiring the Robotic Arm1, X) Setting Up the Robotic Arm 1, Y) Wiring the Robotic Arm 2, Z) Setting Up the Robotic Arm2.

Fig. 1 Project Network for Bulb Production



Critical path technique is useful for finding out the longest path of planned activities which are necessary in order to determine the minimum time required for project completion. There are three distinct paths describing the project: path 1) A-W-X-Y-Z-J-K-L-M-N-O, path 2) A-Q-R-S-T-J-K-L-M-N-O and path 3) A-B-C-D-E-F-G-H-I-J-K-L-M-N-O. Each path length indicated in the following table 1 that can be obtained by estimating the time t_j. Of the three paths, path 3 is the critical path that requires the longest time for the completion of the project. The activities along critical path 3 are used to determine the completion time for the project.

Table 1: Path Length Calculation of Different Paths

Path	Estimated Time	Path length (min)
1.	28.16+ 5.55+2.97+ 3.28+ 9.93+38.69+ 18.7 +12.75+ 68.25+ 125.69 +183.23	8.29
2.	28.16+3.28+ 6.63+9.93+ 4.32 +38.69+ 18.7 +12.75+ 68.25+ 125.69 +183.23	8.32
3.	28.16+102.5+93.75+99.53 +26.5 +45.78+ 130.29 +12.5 +32.25+ 38.69+ 18.7 +12.75+ 68.25+ 125.69 +183.23	16.97

The possible path for this project is 3. So, path 3 is the critical path and path 1 & 2 is negligible according to the path length.

In this particular lamps producing industry General Lighting Service (GLS) is a semi-automated project. The business leader of the company appointed an engineering manager to complete the GLS project within assigned time and assigned a project manager who is responsible for satisfying the business leader’s deadline requirements while staying within the assigned budget. The project manager wanted to crash the activities to shorten the project duration and maintain non-conformance risk activity rate for all project activity. Table 2 represents the activity list of GLS. There are 23 activities. The name and denotation of the activities are given in table 2. The precedence relationships among the activities are also shown.

Table 2: Description of the Activities

Activity	Description	Predecessor activity ,i
A	Flare Making	-
B	Stem Making	A
C	Annealing Oven	B
D	Mount Making	C
E	Marking & Shell	D
F	Sealing	E
G	Pumping	F
H	Cap Filler	G
I	Cap Rail	H
J	Capping Mill	I
K	Flashing	J
L	Cap Soldering	K
M	Repairing	L
N	Pick Up	M
O	Glow Test	N
Q	Stem Machine Installation	A
R	Mount Machine Installation	Q
S	Sealing Machine Installation	R
T	Capping Mill Installation	S
W	Wiring the Robotic Arm1	A
X	Setting Up the Robotic Arm1	W
Y	Wiring the Robotic Arm2	X
Z	Setting Up the Robotic Arm2	Y

Table 3 presents data related to the direct and indirect costs for each activity in the GLS project. The direct cost (c_j or c'_j) of each activity in Table 4 includes expenses related to labor, materials, and equipment. The indirect costs ($ic_{j,}$ ic'_j) includes expense related to salary of the supervisor, salary of the inspectors, security, maintenance. The project manager is required to prepare a practical and cost-effective project schedule that would result in the successful completion of the project by the due date.

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Table 3: Data for Cost for Normal & Crashed Condition

Activity	Normal Labor cost, tk.	c_j ,	Crashed labor cost, tk.	c'_j,	ic_j, tk.	ic'_j, tk.
A	0.39	1.45	0.42	1.57	10.31	9.68
B	1.44	6.91	1.41	7.73	37.52	32.15
C	1.31	1.31	1.42	1.42	34.31	32.50
D	1.39	24.39	1.56	25.1	36.43	35.63
E	0.37	30.37	0.40	32.9	9.70	9.15
F	0.64	0.64	0.69	0.69	16.76	15.83
G	1.82	17.82	1.97	18.9	47.69	45.18
H	0.18	14.58	0.19	15.7	4.58	4.32
I	0.45	2.85	0.48	3.08	11.80	11.06
J	0.00	7.40	0.00	8.00	14.16	13.34
K	0.26	0.26	0.28	0.28	6.84	6.46
L	0.18	0.98	0.19	1.04	4.67	4.39
M	1.91	5.11	2.05	5.65	24.98	23.50
N	3.52	3.52	3.79	3.79	46.00	43.35
O	2.57	2.57	2.77	2.77	67.06	63.42

Table 4: Data for Time for Normal & Crashed Condition

Activity	t_j , sec	t'_j , sec	r_j, sec
A	28.16	26.45	1.71
B	102.5	87.85	14.65
C	93.75	88.81	4.94
D	99.53	97.35	2.18
E	26.5	24.99	1.51
F	45.78	43.26	2.52
G	130.29	123.4	6.86
H	12.5	11.79	0.71
I	32.25	30.22	2.03
J	38.69	36.46	2.23
K	18.7	17.65	1.05
L	12.75	12	0.75
M	68.25	64.2	4.05
N	125.69	118.4	7.24
O	183.23	173.2	9.94
Total Time (min)	16.97	15.93	1.04

Table 4 presents time related to normal & crashed condition of the activities. The indirect activities included related to reduced-time for the supervisor, salary of the inspectors, security and maintenance

5.2 PQLC Estimation

A systematic process for PQLC estimation that includes procedures, scale, and definition is divided into three steps: nonconformance risk identification and coding for project activities, nonconformance risk analysis for project activities, and PQLC estimation with nonconformance risk activity rate (Kim, Kang & Hwang 2012).

Table 5: Code and Description of Nonconformance Risks

Code	Description of conformance risk	Code	Description of conformance risk	Code	Description of conformance risk
A1	Flare crack	F1	Bad shell sealing	K1	Oxidized
A2	Dimension change	F2	Flare cracking	K2	Improper intensity of light
A3	Rough flare surface	F3	Bad shape	K3	Defect in flashing point
B1	Single Lead-in-wire	G1	Button crack	L1	Bad soldering
B2	Exhaust tube broken	G2	Wiping fault	L2	Pin missing
B3	Flange crack	G3	Wire burn	L3	Broken eyelet
C1	Uneven belt line	H1	Improper shape of cap	M1	Drop cap
C2	Improper entry of stem	H2	Defect of construction in cap	M2	Open wire in cap
C3	Stem displacement	H3	Inaccurate cap receiving	M3	Cap fitting problem
D1	Moly wire missing	I1	Uneven rail	N1	Cracked bulb
D2	Bent Lead in wire	I2	Obstacles in path of rail	N2	Broken bulb
D3	Coil loose of bulb	I3	Inappropriate quantity of cement	N3	Faulty in inspection
E1	Bad marking of shell	J1	Long wire	O1	Discontinuous lightning
E2	Drop shell	J2	Bent pin	O2	Unable to lightning
E3	Bad stamping of shell	J3	Inadequate quantity of heating	O3	Less intensity in light

Step 1: Non-Conformance Risk Identification and Coding

A nonconformance risk is defined as any uncertainty that would negatively affect project activity cost if it occurs after an activity is finished. Three nonconformance risks for each activity in table 5 are identified through brainstorming among experts and stakeholders involved in the GLS project.

Step 2: Non-Conformance Risk Analysis for Project Activities

The probability and impact of each nonconformance risk identified in Step 1 have been assessed through interviews with knowledgeable and experienced project team member(s) or expert(s) from outside the project. The previous experiences of experts may be helpful for probability and impact assessment (Scott-Young & Samson 2008). For qualitative analysis, the probability and impact of the nonconformance risk have been assessed with numerical scales: 0.10, 0.30, 0.50, 0.70, and 0.90. The numerical scales are defined in Tables 6 and 7, respectively (Kim, Kang & Hwang 2012). The numerical scales of Tables 6 and 7 are used to develop the probability and impact matrix of Table 8. This matrix specifies combinations of probability and impact that lead to scoring each nonconformance risk identified in step 1 and is used to prioritize nonconformance risks. Numeric values in Table 8 are derived from the nonconformance risk score; (NRS) = probability (P) × impact (I). The probability and impact matrix is useful at the beginning of nonconformance risk analysis when an assessor has limited information about the risks associated with an activity. The nonconformance risk analysis for project activities can be a rapid and cost-effective means to prioritizing before the PQLC estimation (Kim, Kang & Hwang 2012).

Step 3: PQLC Estimation with Nonconformance Risk Activity Rate

The PQLC for nonconformance risk activity was estimated under the assumption that the rework or modification costs for a nonconformance risk activity are equivalent to its crash direct cost c′j. The acceptable number of nonconformance risk activities is decided using Eq. (5). The GLS project PQLC is computed by inserting the nonconformance risk activities of related codes from Table 9 into the proposed modified model formula For this project, the nonconformance risk activity rates from 10% to 50% are given in increments of 10%, and the PQLCs for each nonconformance risk activity rate candidate is estimated at 10%, 20%, 30%, 40%, 50% respectively.

Table 6: The Scale and Definition of Non-Conformance Risks

Probability scale	Definition (based on occurrence of rework or modification after completion of an activity)
0.10	Rare
0.30	Possible
0.50	Likely possible
0.70	Highly likely
0.90	Almost certain

Source: (Kim, Kang & Hwang 2012)

Table 7: The Scale and Definition of Impact Related To Activity Cost

Impact scale	Definition
0.10	<20% increase in activity cost
0.30	20-40% increase in activity cost
0.50	40-60% increase in activity cost
0.70	60-80% increase in activity cost
0.90	>80% increase in activity cost

Source: (Kim, Kang & Hwang 2012)

Table 8: Probability and Impact Matrix

Impact	Probability				
	0.10	0.30	0.50	0.70	0.90
0.10	0.01	0.03	0.05	0.07	0.09
0.30	0.03	0.09	0.15	0.21	0.27
0.50	0.05	0.15	0.25	0.35	0.45
0.70	0.07	0.21	0.35	0.49	0.63
0.90	0.09	0.27	0.45	0.63	0.81

Table 9: Probability for Each Non-Conformance Risk activity

Code	Probability	Code	Probability	Code	Probability
A1	0.62	F1	0.54	K1	0.34
A2	0.21	F2	0.34	K2	0.18
A3	0.17	F3	0.12	K3	0.48
B1	0.50	G1	0.60	L1	0.30
B2	0.20	G2	0.10	L2	0.46
B3	0.30	G3	0.30	L3	0.24
C1	0.32	H1	0.14	M1	0.18
C2	0.20	H2	0.18	M2	0.42
C3	0.48	H3	0.68	M3	0.40
D1	0.17	I1	0.51	N1	0.32
D2	0.59	I2	0.10	N2	0.32
D3	0.24	I3	0.39	N3	0.36
E1	0.34	J1	0.35	O1	0.40
E2	0.17	J2	0.45	O2	0.20
E3	0.49	J3	0.20	O3	0.40

The individual probability for Each Non-Conformance risk is shown in Table 9. To formulate a priority ranking of the individual codes based on NRSs, the NRSs are rearranged in descending order in Table 10. If there is more than one code in the same ranking row of Table 10, priority is given to the nonconformance risk code of the activity with a relatively high crash direct cost of them.

Table 10: Priority ranking of individual codes

Ranking	Code	NRS	Ranking	Code	NRS
1	G1	0.54	16	I3	0.19
2	D2	0.53	17	B2 , J3	0.18
3	B1	0.45	18	L3	0.17
4	A1	0.43	19	D1,E3,A2,C3	0.15
5	J2	0.41	20	K3	0.14
6	F1	0.38	21	M1	0.13
7	J1, L2	0.32	22	A3	0.12
8	M2	0.29	23	N3	0.11
9	M3	0.28	24	E1, O2, K1,	0.10
10	G3, B3	0.27	25	N1,N2,G2,C1	0.09
11	I1	0.26	26	F3	0.08
12	F2	0.24	27	C2	0.06
13	D3	0.22	28	I2,E2,K2,H2	0.05
14	H3, L1	0.21	29	H1	0.04
15	O1, O3	0.20			

6. Results

The direct cost of each activity includes expenses related to labor, materials, and equipment and indirect costs include expenses related to salary of the supervisor, salary of the inspectors, security, and maintenance. The computed direct cost per unit time for each activity, indirect cost per unit time for each activity, additional direct cost when activity is completed in crash time, additional indirect cost when activity is completed in crash time and total project completion cost are given in table 11.

Table 11: Incremental and Decremented Cost of Crashing Activity

Activity	m_j, tk./sec	b_j, tk./sec	E, tk.	F, tk.	T, tk.
A	0.07	0.19	0.12	0.33	6.80
B	0.06	0.19	0.83	2.81	26.38
C	0.02	0.19	0.11	0.95	19.13
D	0.41	0.19	0.90	0.42	44.70
E	1.67	0.19	2.53	0.29	35.41
F	0.02	0.19	0.05	0.48	9.34
G	0.17	0.19	1.15	1.32	42.58
H	1.71	0.19	1.21	0.14	16.95
I	0.11	0.19	0.23	0.39	8.98
J	0.27	0.19	0.60	0.43	14.75
K	0.02	0.19	0.02	0.20	3.81
L	0.08	0.19	0.06	0.14	3.40
M	0.13	0.19	0.54	0.78	18.08
N	0.04	0.19	0.27	1.39	27.40
O	0.02	0.19	0.21	1.91	37.38
Total cost (Tk.)			8.84	11.90	315.10

6.1 Comparison between Existing Model and Modified Model

To compute the total project crashing cost, the normal project completion cost is added to the extra cost for crashing activities. The normal project completion cost is 315 tk. per project.

Existing Model Calculation:

The normal project completion cost = 315.10 tk.
Increased direct cost = 8.84 tk.
PQLC = 53.40 tk.

The total additional direct cost = 62.24 tk.

Total project direct crashing cost = (Normal project completion cost + Additional direct cost)
= (Normal project completion cost + Increase in direct cost + PQLC)
= (315.10 tk. + 8.84 + 53.40) tk.
= 377.34 tk.
The crashing cost considering direct cost and PQLC have been calculated by the existing model.

Proposed Model Calculation:

The normal project completion cost = 315.10 tk.
Increased direct cost = 8.84 tk.
PQLC = 53.40 tk.
The total additional direct cost = 62.24 tk.
Total additional indirect cost = 11.90 tk.

Total project crashing cost = (Normal project completion cost + Additional direct cost – Additional indirect cost)
= (Normal project completion cost + Increase in direct cost + PQLC – Decrease in indirect cost)
= (315.10 Tk. + 8.84 + 53.40 - 11.90) tk.
=315.10 tk. + 50.34 tk.
= 365.44tk.
The crashing cost considering direct and indirect cost has been calculated by the proposed model and that is 50.34 tk.

If crashing is considered in a project then total project cost will be increased from 315tk. to 377.34 tk. per project where only direct cost is considered. But the total cost will be increased by 365.44 tk. per project where direct and indirect both are considered.
On the other side, the normal completion time is 16.97 min and the completion time after crashing is 15.93 min. In this case, 1.04 min has been reduced for a project under crashing condition.

6.2 Estimated PQLC

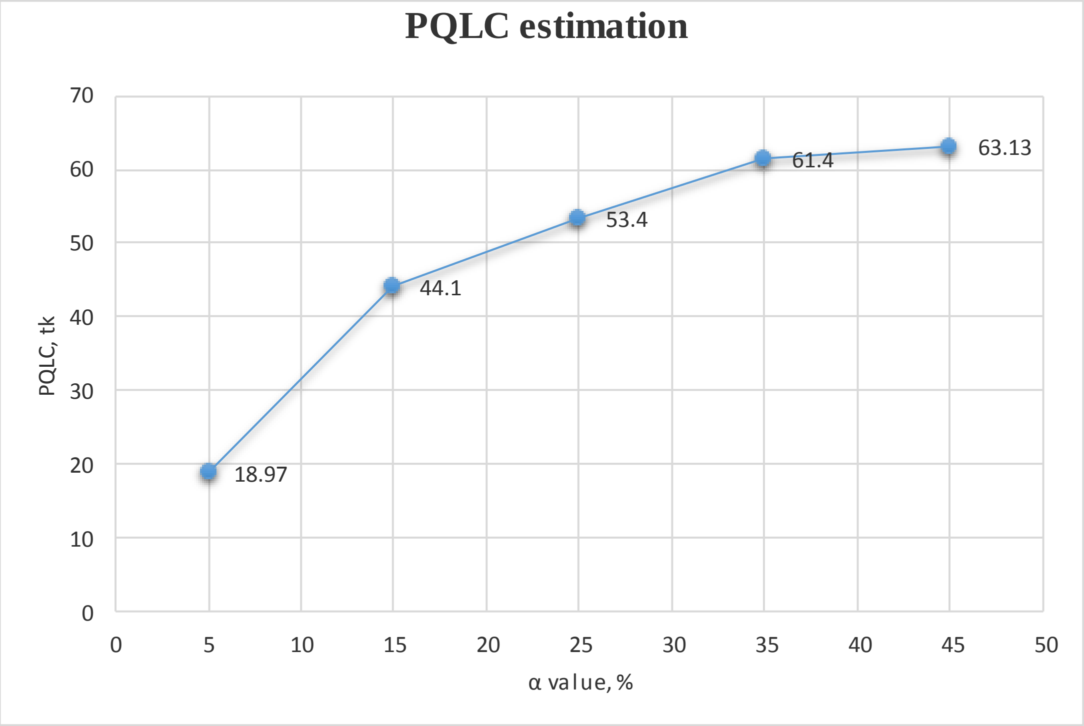
Table 12 presents each optimal PQLC for the eight non-conformance risk activity rate candidates. The nonconformance risk activities shown in Table 11 had higher NRS than the other activities. The GLS project PQLC is directly related to α value or to the number of nonconformance risk activities. Although the RTPSI project requires excessive crashing for nonconformance risk activities, cases in which preventive measures against rework or modifications can be taken allow the project manager to choose a low α value without concern about the PQLC. The information presented in Table 12 and Fig. 3 allowed the project manager to complete the project within the assigned budget and due date in this real-life case.

Table 12: PQLC for Non-Conformance Risk Activity Rate Candidate

α , %	5%	15%	25%	35%	45%
Z_j	1	2	4	5	7
Non-conformance	G	G, D	G, D, B, A,	G, D, B,A,J	G, D, B, A, J, F, L
PQLC, tk	18.97	44.10	53.40	61.40	63.13

The potential quality loss cost (PQLC) is visualized in Fig. 2. If no rework or modification occurs while executing nonconformance risk activities, the PQLC is saved. If any rework or modifications occur during the execution of these activities, the PQLC is payable in the budget. (J. Kim et al., 2012).

Fig 2: PQLC Increase Related to α Value



The results in this paper are new and none of the previous research has found this result. The result of this research supports the previous theory and additionally considered indirect costs that minimized the total cost of the project. This paper has introduced indirect crashing cost which has not been incorporated in any other previous research.

7. Discussion

A project is a one-time task constrained by time, cost, and quality. In this paperwork, a mixed integer linear programming model has proposed that takes into account the direct and indirect cost for crashing activities including potential quality loss cost. In this model, the rework and modification time and cost are bounded by crash duration and crashing direct cost respectively of the activity. The data of normal activity cost, normal completion time, crashing cost, crashing time of an industry have been collected to calculate the direct and indirect crashing cost. The total cost considering PQLC are minimized under some assumptions. The problem has been solved by LINDO software. The objective function is the total cost for crashing activities. A comparison between existing model and modified model is shown. In the real-life situation, 1.04 min has been reduced for a project under crashing condition and the cost increases 15% for crashing. The existing model gives a practical solution and precise calculation of cost distribution can be determined.

8. Conclusion

In this paper, a mixed integer linear programming model has been proposed, considering the total crashing cost that accounts for both the non-conformance risk and PQLC of the project activities. Unlike the traditional time-cost tradeoff, here it has been proposed that the project quality should also be taken into consideration along with the direct and indirect cost in the time-cost tradeoffs. Project manager limits the number of non-conformance risk activity. So the PQLC estimation becomes easier to calculate. A practical and valid mathematical model is proposed that indicates project completion before due date within minimal total cost. The models developed in this paperwork could be easily extended to accommodate nonlinear relationships. This modified mathematical model takes into consideration the real situation a project. It is also important for automation in production line under time-limited projects The main objective of this research work is to include quality loss cost for excessive crashing activity and also the indirect cost for crashing activities of a project has been satisfied. This research work has also fulfilled another objective to apply the modified mathematical model to a real-life problem to test its feasibility. The result of this research supports the previous theory and additionally considered indirect costs that minimized the total cost of the project. This paper has introduced indirect crashing cost. It also inaugurated about crashing activities, direct crashing cost & potential quality cost (PQLC). In the modified model cost is reduced by the existing model which helps in decision-making. The completion of the project before due date within minimal total cost is possible. It is a valid and practical model that can be used in a similar type of project.

This proposed mathematical model is formulated under the unlimited resource. But in modern scheduling problems, the resource-constrained scenario is found. One of the assumptions of the proposed model is that the time-cost tradeoffs are linear. But actually, there is very few evidence that time-cost relation is linear. It is assumed linear for the simplicity of the calculation. In future research, the proposed model may be extended estimating the uncertainty of nonconformance risk activities with the additional cost of uncertain value.

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